Modeling Parameter Interactions in Ranking SVM

Yaogong Zhang, Jun Xu, Yanyan Lan, Jiafeng Guo, Maoqiang Xie, Yalou Huang, Xueqi Cheng
1 College of Computer and Control Engineering, Nankai University
2 Institute of Computing Technology, Chinese Academy of Sciences

1. Pairwise learning to rank: ranking as binary classification over preference pairs

Motivation: There exist significant interactions among the training pairs, e.g., (doc1, doc2) and (doc1, doc3) share doc1. Whether there also exist interactions among model parameters? How to utilize the interactions if the answer is yes?

2. Parameter interactions in Ranking SVM

Ranking SVM

Prime
\[ \min_{w \in \mathbb{R}^n} \frac{1}{2} ||w||^2 + C \sum_{(i,j) \in P} \left[ 1 - \langle w, x_i - x_j \rangle \right]_+ \]

Dual
\[ \min_{a} \frac{1}{2} a^T M a - e^T a \]
\[ \text{s.t.} \quad 0 \leq a_{ij} \leq C, \forall (i,j) \in P \]

Low rank structure among Lagrange multipliers \( a_{ij} \)

3. Factorized Ranking SVM

Directly modeling the low rank structure: \( a_{ij} = \langle v_i, v_j \rangle \)

\[ \alpha_{ij} \]
\[ V \]
\[ V^T \]

New loss function
\[ \min_{v_1, \cdots, v_N} \left\{ \sum_{(i,j) \in P} \left[ (v_i, v_j)(x_i - x_j) \right] \right\}^2 + \]
\[ C \sum_{(k,l) \in P} \left[ 1 - \sum_{(i,j) \in P} (v_i, v_j)(x_i - x_j, x_k - x_l) \right]_+ \]

Number of parameters: \( O(N^2) \rightarrow O(KN) \)

4. Experiments

- Results on OHSUMED (dense preference pairs)
  - MAP
    - RSVM: 0.4427
    - RankNet: 0.404
    - ListNet: 0.4443
    - Fac-RSVM: 0.4623

- Results on MQ2008 (sparse preference pairs)
  - MAP
    - RSVM: 0.4713
    - RankNet: 0.4522
    - ListNet: 0.4806
    - Fac-RSVM: 0.4714

5. Conclusion

1. There exists a low-rank structure among the Lagrange multipliers of Ranking SVM.
2. Factorized Ranking SVM decomposes each Lagrange multiplier as a dot product of two low-dimensional vectors.
3. Factorized Ranking SVM decreases space complexities from \( O(N^2) \) to \( O(KN) \).
4. Experimental results showed that Factorized Ranking SVM outperformed all baselines.

1) Factorized Ranking SVM outperformed all baselines including Ranking SVM.
2) More improvements can be achieved on datasets with denser preference pairs.