Stochastic Rank Aggregation

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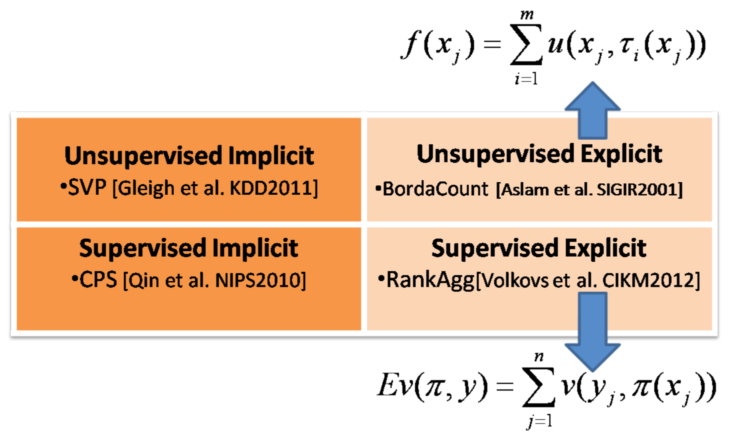
1. Rank Aggregation Problem

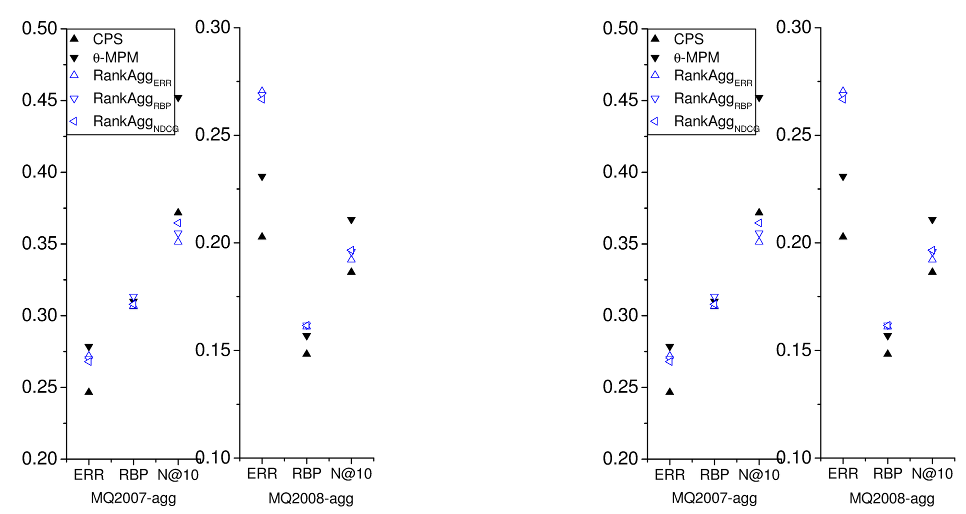
- Crucial to many applications
  - meta search, collaborative filtering
- Formalization of Two common scenario
  - Unsupervised Rank Aggregation
    Given $\tau_1, \ldots, \tau_m$, we are to find a consensus ranking π that
    $$\max \sum_{i=1}^{m} \mathcal{M}(\pi, \tau_i),$$
    where $\mathcal{M}$ is the similarity measure between two rankings.
  - Supervised Rank Aggregation
    Given $\{D^{(i)}\}_{i=1}^{N}$, and all ranking inputs $\tau_1^{(i)}, \ldots, \tau_m^{(i)}$ over $D^{(i)}$, $y^{(i)}$ is the ground truth ranking over $D^{(i)}$, an aggregation function f can be learned by
    $$\max \sum_{i=1}^{N} \mathcal{M}(y^{(i)}, f(\tau_1^{(i)}, \ldots, \tau_m^{(i)})).$$
- Implicit and Explicit Methods

  whether rank information is used directly:



2. Motivation

Failure of Explicit Methods:

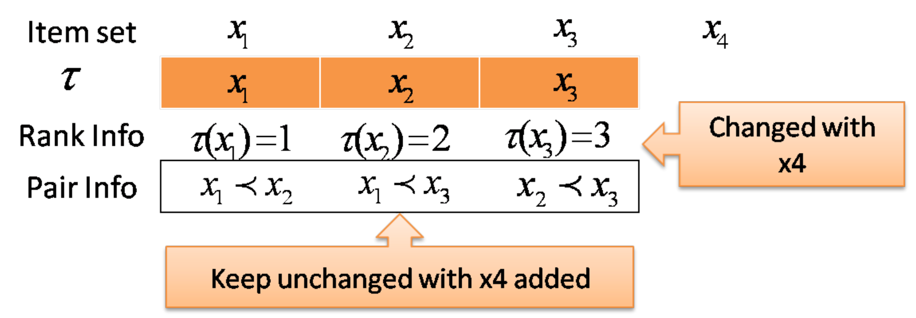


Unsupervised Supervised

Underlying Reason: Unreliable rank information from partial rankings

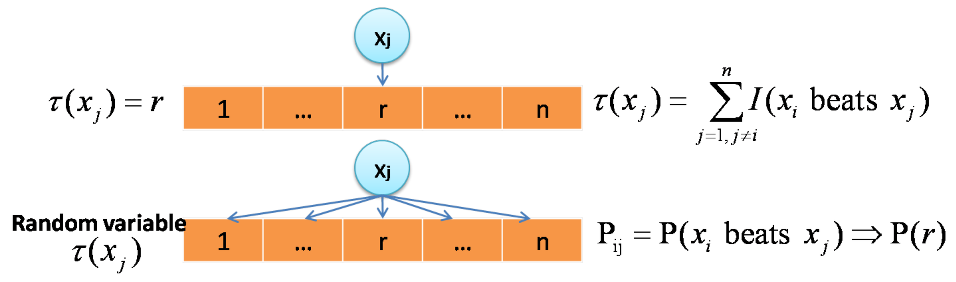
3. Stochastic Rank Aggregation

Our Observation: pairwise information is more robust to incompleteness.



Our Solution: Incorporate uncertainty into rank aggregation in a pairwise way

- Rank to Rank Distribution



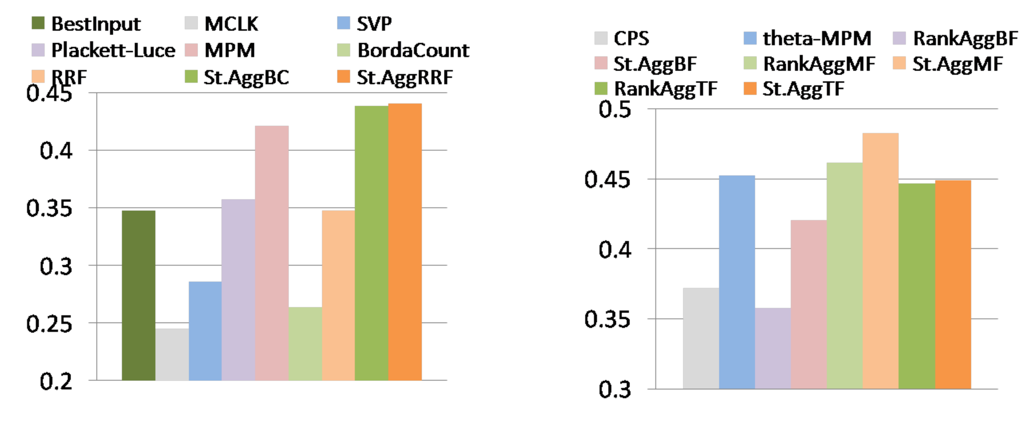
When the new item x_i comes, the rank distribution of x_j, $\mathrm{P}(r)$ can be updated as:

$$\mathrm{P}^{(t)}(r) = \mathrm{P}^{(t-1)}(r-1)\mathrm{P}_{ij} + \mathrm{P}^{(t-1)}(r)(1-\mathrm{P}_{ij})$$

- Stochastic Rank Aggregation (St.Agg) in Unsupervised Scenario
  - Definition of P_{ij}:
    - ∗ Encode rank difference information
    - ∗ e.g.
    $$\mathrm{P}_{ij} = \begin{cases} \frac{\tau(x_j)-\tau(x_i)}{n}, & \text{if} \tau(x_i) < \tau(x_j) \\ 1 - \frac{\tau(x_i)-\tau(x_j)}{n}, & \text{if} \tau(x_j) < \tau(x_i) \\ 0.5, & \text{otherwise} \end{cases}$$
  - Expectation: New Ranking Function
    $$f_s(x_j) = \sum_{i=1}^{m} \sum_{r=1}^{n} u(x_j, \tau_i(x_j))P(r)$$
- Stochastic Rank Aggregation (St.Agg) in Supervised Scenario
  - Definition of P_{ij}:
    - ∗ assume $s_j \sim \mathcal{N}(f(x_j), \sigma^2) \Rightarrow s_i - s_j \sim \mathcal{N}(f(x_i)-f(x_j), 2\sigma^2) \Rightarrow \mathrm{P}_{ij} = \mathrm{P}(s_i - s_j > 0)$
  - Expectation:
    - ∗ New Optimization Objective Functions, e.g.
    $$Ev_s(\pi, y) = \sum_{j=1}^{n} \sum_{r=1}^{n} v(y_j, r)P_j(r)$$
    - ∗ Feature-based optimization framework
      1. Feature Mapping Design: (1)item Ψ_{BF};(2)item-item Ψ_{MF};(3)user-item-item Ψ_{TF};
      2. Learning Method: Gradient Method

4. Experiments

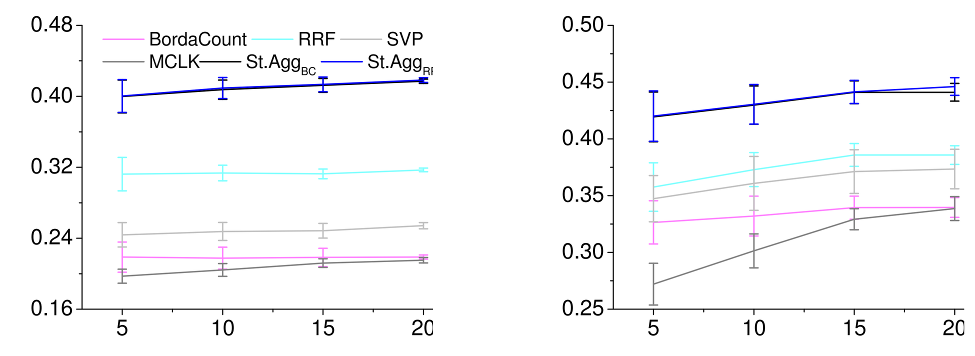
- Datasets: metasearch tasks in LETOR4.0
  - e.g. MQ2007-agg and MQ2008-agg
- Effectiveness: e.g. NDCG@10 on MQ2007-agg



Unsupervised Supervised

1. St.Agg performs better than baselines in both scenarios;
2. The item-item feature Ψ_{MF} performs best in supervised scenario.

- Robustness: e.g. NDCG@5 of St.Agg



MQ2007-agg MQ2008-agg

St.Agg stays above baselines however the ranking inputs change.

5. Conclusion & Future Work

Conclusion:

1. We find that unreliable rank information from partial ranking inputs will make the approaches directly using rank information fail in practice;
2. We propose a novel rank aggregation framework by incorporating uncertainty to tackle this problem;
3. The framework turns out to be effective and robust.

Future work:

1. To find better ways to define rank distributions;
2. To utilize this stochastic rank aggregation framework in other applications.